

## Calculus 12: Test 3 Outline and Review

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### You should be able to:

1. Differentiate various types of functions including trigonometric, exponential and logarithmic functions.
2. Solve various related rates problems, including problems with trigonometry.
3. Solve various application questions.
4. Determine critical points and inflection points.
5. Determine intervals for which a function is increasing, decreasing, concave up and concave down.
6. Differentiate both explicitly and implicitly.
7. Simplify expressions that have been differentiated.
8. Solve profit/revenue optimization problems.
9. Solve optimization problems involving surface area or volume, and distance.

### Review Questions:

1. Differentiate each of the following expressions with respect to  $x$ . Isolate  $dy/dx$ .

a)  $y = x^3 - 4x + \sqrt{2x}$

b)  $y = (2x - 5)^2(x + 3)^3$

c)  $y = \frac{5 - 3x}{x^2 - 2}$

d)  $y = \sin^2(4x)$

e)  $xy^2 + 3y^2 = x$

f)  $y = \tan(x^2 - 3)$

g)  $\sin^2(x) = y^3 + 5x$

h)  $y = x \cos(3x)$

i)  $y = 2xe^{3x}$

j)  $y = \frac{\ln(3x)}{2x - 4}$

k)  $y = \log_4(\sin(5x))$

l)  $y = \frac{4^{2x-5}}{\tan(2x)}$

2. Differentiate each of the following with respect to the indicated variable and simplify.

(a)  $y = \cos(3x + 5); x$

(b)  $y = \ln^3(x^2 - 4x + 1); x$

(c)  $y = 3\sqrt{\tan(2x)}; x$

(d)  $y = x^2(e^{2x}); x$

(e)  $\sin(2\theta - 5) + w = 5; \theta$

(f)  $y = 3x^{2x}; x$

3. Determine the interval for which  $y = 5x^2e^{2x}$  is decreasing.

4. Determine the interval for which  $y = 3\ln(x^2 + 4)$  is concave up.

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5. An airplane, flying at 450 km/hr at a constant altitude of 5 km, is approaching a camera mounted on the ground. Let  $\theta$  be the angle of elevation above the ground at which the camera is pointed. When  $\theta = \frac{\pi}{3}$ , how fast does the camera have to rotate in order to keep the plane in view?
6. We want to construct a box whose base length is 4 times the base width. The material used to build the top and bottom cost twice as much as the material used to build the sides. If the box must have a volume of  $200 \text{ cm}^3$  determine the dimensions that will minimize the cost to build the box.
7. A certain species of bird nests on an island A 5 km from the nearest point B of the straight mainland shoreline. These birds are released from a banding point C on the shoreline which is 13 km from point B. The birds instinctively choose to fly partly along the shoreline to point D and then across the water in such a way as to minimize their energy expenditure. They use 1.3 times as much energy to fly across water as they do to fly across land. How far is it from C to D?
8. Determine all critical points and inflection points for the function  $y = x^2e^{2x}$ . Sketch the function labeling the critical points and inflection points.
9. Determine the min/max point(s) for  $y = xe^{-x}$ .
10. The temperature,  $T$ , in degrees Fahrenheit, of a cold yam placed in a hot oven is given by  $T(t)$ , where  $t$  is the time in minutes since the yam was put in the oven.
  - i) What is the sign of  $T'(t)$ ? Explain.
  - ii) What are the units of  $T'(20)$ ?
  - iii) What is the practical meaning of the statement  $T'(20) = 2$ ?